

GRAVITATIONAL COLLAPSE OF A RADIATING SHELL AND RADIATION BURSTS

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We study the collapse of a self-gravitating thick shell of bosons coupled to a scalar radiation field. Due to the non-adiabaticity of the collapse, the shell (quantum) internal degrees of freedom absorb energy from the (classical) gravitational field and are excited. The excitation energy is then emitted in the form of bursts of (thermal) radiation and the corresponding backreaction on the trajectory is estimated.

It is clear that classical physics is not sufficient for a complete description of the gravitational collapse: firstly, the collapse ending into a point-like singularity is forbidden by the uncertainty principle; secondly, the backreaction of semiclassical effects (with matter evolving quantum mechanically on a classical space-time) such as the Hawking radiation¹ should be properly included². We have investigated the semiclassical limit for various models^{3,4} by employing a Born-Oppenheimer decomposition of the corresponding minisuperspace wavefunction⁵. In particular⁶, we considered the collapse in vacuum of a “macroshell” made of N quantized “microshells”, each of which corresponding to a typical hadronic mass $m \sim 1 \text{ GeV}$ s -wave scalar particle, with the proper mass of the macroshell $M = Nm$. Due to the bosonic nature of the microshells, the whole system formed a “condensate” bound by their mutual gravitational attraction and it was then shown that the collapse induced non-adiabatic transitions from the ground state to higher excited states. Once enough microshells were excited, they could collectively decay back to the ground state by creating additional particles, the whole process resulting in a transformation of gravitational energy into matter. However, such a model is unrealistic because the absence of signals from the shell precludes observations, hence an effective minisuperspace action for radiating shells was derived⁷ as a key tool for making observational predictions^{8,9}.

We order the microshells according to their area ($r_1 < r_2 < \dots < r_N$) and assume the thickness of the macroshell $\delta = r_N - r_1$ is small ($\delta \ll r_1$). The space between two microshells has Schwarzschild geometry with mass function M_i which vanishes for $r < r_1$ and equals the total ADM mass $M_N \equiv M_s$ for $r > r_N$. We also assume the microshells start out at large radii with negligible velocity, hence $M_{i+1} = M_i + m$, and use the standard junction equations¹⁰. It is then convenient to consider each microshell in the mean field of the others (Hartree approximation^{11,6}) which yields a classical effective Hamiltonian for each microshell⁹ $H_m = (1/2)m\dot{\bar{r}}^2 + V_m$, where an overdot denotes the derivative with respect to the microshell proper time τ , $\bar{r} = r - R$ is the displacement of the microshell relative to the average radius R and, to leading order in $|\bar{r}|/R$ and m/M ,

$$V_m \simeq \begin{cases} \frac{G M_s m}{2R} \frac{\bar{r}}{R} \left(1 - \frac{G M^2}{2R M_s}\right) & \bar{r} > +\frac{\delta}{2} \\ \frac{G M_s m}{2R^2} \left(\frac{\bar{r}^2}{\delta} + \frac{\delta}{4}\right) - \frac{G^2 M^2 m}{4R^3} \bar{r} & |\bar{r}| \leq \frac{\delta}{2} \\ \frac{G M_s m}{2R} \frac{\bar{r}}{R} \left(-1 - \frac{G M^2}{2R M_s}\right) & \bar{r} < -\frac{\delta}{2} \end{cases} \quad (1)$$

The potential V_m accounts for tidal effects (backreaction) between the microshells and, for $R > R_H \equiv 2GM_s$, confines the microshells around $\bar{r} = 0$ within the thickness ⁶

$$\delta \sim \ell_m^{2/3} R^{1/3} \left(\frac{R}{R_H} \right)^{1/3}, \quad (2)$$

where $\ell_m = \hbar/m$ is the Compton wavelength of a microshell.

We can now quantize the microshells and obtain a Schrödinger equation with explicit time-dependence due to $R = R(\tau)$ in the potential,

$$i \hbar \dot{\Phi} = \left[\frac{\hat{\pi}_r^2}{2m} + \hat{V}_m \right] \Phi. \quad (3)$$

One can solve this equation by making use of invariant operators ¹² and then compute the transition amplitudes $A_{0 \rightarrow n}(\tau)$ from the ground state $\Phi_0(0)$ to a state $\Phi_n(\tau)$ with (higher) energy $E_n = n \hbar \Omega$, where $\Omega = (1/R) \sqrt{R_H/2\delta}$. Let us also note that, since the widths of the lowest states are of order δ , the bosonic microshells are essentially superimposed and form a condensate ⁹. To lowest order in \dot{R} , one finds $A_{0 \rightarrow 0} \simeq 1$, $A_{0 \rightarrow 2n+1} = 0$ and

$$A_{0 \rightarrow 2n}(\tau) \simeq (-i)^n \frac{\sqrt{(2n)!}}{3^n 2^{n/2} n!} \left(\frac{\delta}{R_H} \right)^{n/2} \dot{R}^n. \quad (4)$$

We remark that the above transition amplitude is a consequence of both a quantum mechanical non-adiabatic effect ($A_{0 \rightarrow 2n} \propto \dot{R}^n$) and the finite thickness of the macroshell ($A_{0 \rightarrow 2n} \propto \delta^{n/2}$), the latter being further related to the quantum mechanical nature of the model ($\delta \propto \hbar^{2/3}$). The above expression is, for realistic cases, small, and the probability for a microshell to get excited twice during the whole collapse is therefore negligible.

Since the proper mass of each microshell has a non-vanishing probability to increase in time, the collapse is not a free fall: the trajectory of the macroshell slows down ⁶, and this effect is further sustained by the emission ⁷ which quickly brings the proper mass back to the initial value. We shall then consider M as effectively constant along the collapse and compute the net variation of M_s due to the loss of the (excited state) proper energy in time. The whole process is a transformation of gravitational energy into radiation. Of course, if $\dot{R} = 0$ for $R \sim R_H$, the proper acceleration of the macroshell would equal the surface gravity $a_H = 1/2 R_H$ of a black hole of mass M_s . In this limiting case one could exploit the analogy between Rindler coordinates for an accelerated observer in flat space-time and Schwarzschild coordinates for static observers in a black hole background ¹³ and find that the shell emits the excess energy with Hawking temperature

$$T_H = \frac{\hbar a_H}{2 \pi k_B} = \frac{\hbar/k_B}{8 \pi M_s}, \quad (5)$$

where k_B is the Boltzmann constant. We previously proposed an explicit construction which leads to this result ³, but here we shall determine $R = R(\tau)$ from the equation of motion.

We now consider an isotropic massless scalar field φ conformally coupled to gravity ¹³ and to the microshells via the radiation coupling constant e , and study the emission of quanta of radiation occurring when the microshells in the state Φ_2 decay back into the ground state Φ_0 . For the cases we consider (see Table 1 for an example) the emissions occur practically in phase, since the ratio between the thickness of the shell and the typical wavelength of the emitted quanta is small and the distance covered between each of the

numerous emissions is small, again with respect to the radiation wavelength. This process is therefore coherent and the transition probability per unit Schwarzschild time (dP/dt) can be easily obtained in perturbation theory to leading order in e ⁹.

It is then straightforward to estimate the rate of proper energy lost by the macroshell per unit (Schwarzschild) time,

$$\frac{dE}{dt} = -4\pi R^2 \sum_{\omega} \mu(\omega) \Gamma(\omega) \hbar \omega \frac{dP}{dt}, \quad (6)$$

where $\mu(\omega) = (1 - R_H/R)^{3/2} \omega^2$ the phase space measure for radiation quanta of frequency ω (measured at the shell position $r = R$) and $\Gamma \sim 1$ the gray-body factor for zero angular momentum outgoing scalar waves. The sum in Eq. (6) is dominated by the contribution with $\omega = 2\Omega$ and the flux becomes

$$\frac{dE}{dt} \simeq -\frac{16\pi^2 e^2}{9} \frac{N^2 \ell_m^{8/3}}{R^{10/3} R_H^{4/3}} \left(1 - \frac{R_H}{R}\right) \frac{\dot{R}^2}{e^{2\Omega} \sqrt{1 - R_H/R} / k_B T_H - 1}, \quad (7)$$

in which there appears a Planckian factor with Tolman shifted (instantaneous) Hawking temperature. We note that this feature is not sufficient to relate our model to the usual Hawking effect, since Eq. (7) contains a coupling constant e , while the probability of Hawking emission does not depend on any coupling ¹.

One can now integrate numerically the equation of motion for the (average) radius of the macroshell ¹⁰,

$$\dot{R}^2 = -1 + \left(\frac{M_s}{M}\right)^2 + \frac{G M_s}{R} + \frac{G^2 M^2}{4 R^2}, \quad (8)$$

together with the equation for M_s ⁷,

$$\dot{M}_s \simeq \frac{dE}{dt}, \quad (9)$$

until R is larger than a few times R_H (since our approximations break down for $R \sim R_H$ ^{6,9}). In general, one finds that the non-adiabatic excitations giving appreciable changes in M_s occur relatively close to the horizon and there is no strong dependence on the initial value of R . However, despite the small change in M_s for $R \gg R_H$, we find a large backreaction on the trajectory ⁹ (see Table 1 and Fig. 1 for a typical case). In particular \dot{R} remains (negative and) small (mostly within a fraction of a percent of the speed of light), thus confirming our approximation scheme in which we just kept the leading order in \dot{R} .

To conclude we have seen that the states in a collapsing shell of bosonic matter, initially in free fall, become excited due to quantum non-adiabatic transitions and gravitational energy is lost through the emission of scalar radiation. This effect is intrinsically quantum mechanical, since it is a consequence of the quantum mechanical-bound state nature of the macroshell and the coherence of the emitted radiation. This can cause the shell to lose enough energy so that the backreaction on the trajectory of the radius is large. Since the origin of the studied effect is the non-adiabaticity of the collapse and coherence, one might argue that an analogous phenomenon can happen for all collapsing matter, including the accretion disks around black holes. Our result would thus suggest a new mechanism by which the accreting matter can emit radiation. Whether this mechanism can be related to the Hawking effect is a point which requires further study.

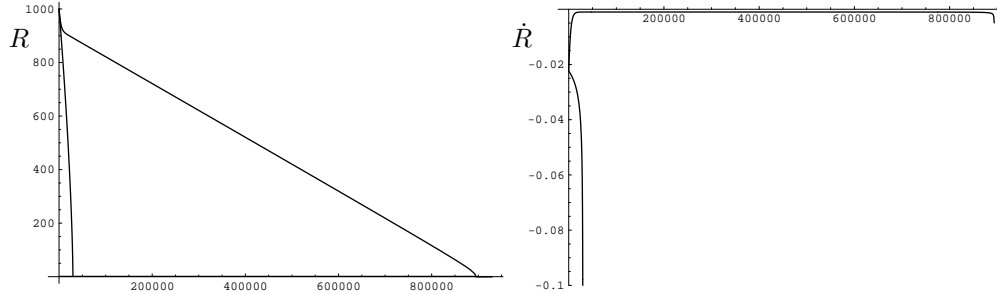


Figure 1. Trajectory $R(\tau)$ and velocity $\dot{R}(\tau)$ of the radiating shell in units of $R_H(0) = 4 \cdot 10^{-12}$ cm for $N = 2 \cdot 10^{40}$ and $e = 8 \cdot 10^{-16}$ (upper curves) compared to the non-radiating collapse (lower curves).

radiation wavelength λ	$10^{-10} \div 10^{-7}$ cm
δ/λ	$10^{-3} \div 10^{-1}$
total number of emissions	10^{39}
N_δ	$10^{35} \div 10^{36}$

Table 1. Typical values of the relevant quantities for the case in Fig. 1. N_δ is the average number of emissions while the shell moves a space δ .

References

1. S.W. Hawking, *Nature* **248**, 30 (1974); *Comm. Math. Phys.* **43**, 199 (1975).
2. P. Hajicek, *Phys. Rev. D* **36**, 1065 (1987).
3. R. Casadio and G. Venturi, *Class. Quantum Grav.* **13**, 2715 (1996).
4. R. Casadio, F. Finelli and G. Venturi, *Class. Quantum Grav.* **15**, 2451 (1998); F. Finelli, G.P. Vacca and G. Venturi, *Phys. Rev. D* **58**, 103514 (1998); G.L. Alberghi, R. Casadio and A. Gruppiso, *Phys. Rev. D* **61**, 124018 (2000).
5. R. Brout and G. Venturi, *Phys. Rev. D* **15**, 2436 (1989); C. Bertoni, F. Finelli and G. Venturi, *Class. Quantum Grav.* **13**, 2375 (1996); R. Casadio, *Int. Jour. Mod. Phys. D* **9**, 511 (1998).
6. G.L. Alberghi, R. Casadio, G.P. Vacca and G. Venturi, *Class. Quantum Grav.* **16**, 131 (1999).
7. G.L. Alberghi, R. Casadio and G. Venturi, *Phys. Rev. D* **60**, 124018 (1999).
8. R. Casadio, *Nucl. Phys. B* (Proc. Suppl.) **88**, 273 (2000).
9. G.L. Alberghi, R. Casadio, G.P. Vacca and G. Venturi, *Gravitational collapse of a radiating shell*, gr-qc/0102014.
10. W. Israel, *Nuovo Cimento B* **44** 1, (1966); *Nuovo Cimento B* **48** 463, (1966).
11. A. Messiah, *Quantum mechanics* (North-Holland Publishing Co., Amsterdam, 1961); S. Schweber, *An introduction to quantum field theory* (Harper and Row, New York, 1961).
12. H.R. Lewis and W.B. Riesenfeld, *J. Math. Phys.* **10**, 1458 (1969).
13. N.D. Birrell and P.C.W. Davis, *Quantum fields in curved space* (Cambridge, Cambridge University Press, 1982)